M13/5/MATSD/SP2/ENG/TZ2/XX

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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 10 May 2013 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *information booklet* is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 19]*

Forty families were surveyed about the places they went to on the weekend. The places were the circus (C), the museum (M) and the park (P).

16 families went to the circus
22 families went to the museum
14 families went to the park
4 families went to all three places
7 families went to both the circus and the museum, but not the park
3 families went to both the circus and the park, but not the museum
1 family went to the park only

(a) Draw a Venn diagram to represent the given information using sets labelled C, M and P. Complete the diagram to include the number of families represented in each region. *[4 marks]*

(b) Find the number of families who

(i) went to the circus only;

(ii) went to the museum and the park but not the circus;

(iii) did not go to any of the three places on the weekend. *[4 marks]*

(c) A family is chosen at random from the group of 40 families. Find the probability that the family went to

(i) the circus;

(ii) two or more places;

(iii) the park or the circus, but not the museum;

(iv) the museum, given that they also went to the circus. *[8 marks]*

Two families are chosen at random from the group of 40 families.

(d) Find the probability that both families went to the circus. *[3 marks]*

2. *[Maximum mark: 17]*

Francesca is a chef in a restaurant. She cooks eight chickens and records their masses and cooking times. The mass m of each chicken, in kg, and its cooking time t, in minutes, are shown in the following table.

Mass m (kg)	**Cooking time t (minutes)**
1.5	62
1.6	75
1.8	82
1.9	83
2.0	86
2.1	87
2.1	91
2.3	98

(a) Draw a scatter diagram to show the relationship between the mass of a chicken and its cooking time. Use 2 cm to represent 0.5 kg on the horizontal axis and 1 cm to represent 10 minutes on the vertical axis. *[4 marks]*

(b) Write down for this set of data

(i) the mean mass, $\overline{m}$;

(ii) the mean cooking time, $\overline{t}$. *[2 marks]*

(c) Label the point $\mathrm{M}(\overline{m}, \overline{t})$ on the scatter diagram. *[1 mark]*

(d) Draw the line of best fit on the scatter diagram. *[2 marks]*

(e) Using your line of best fit, estimate the cooking time, in minutes, for a 1.7 kg chicken. *[2 marks]*

(f) Write down the Pearson's product–moment correlation coefficient, r. *[2 marks]*

(g) Using your value for r, comment on the correlation. *[2 marks]*

The cooking time of an additional 2.0 kg chicken is recorded. If the mass and cooking time of this chicken is included in the data, the correlation is weak.

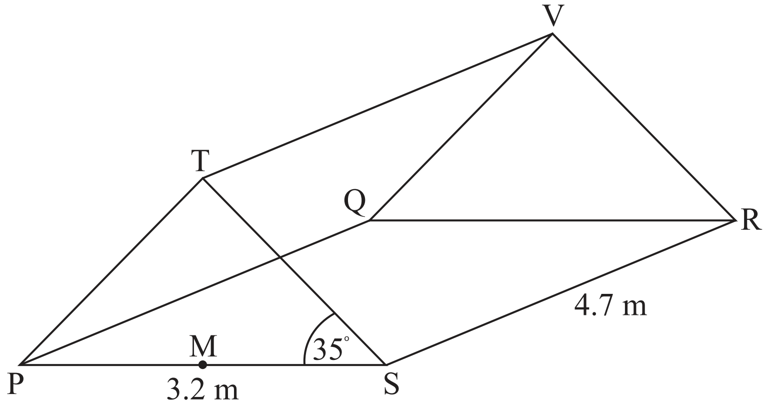
(h) (i) Explain how the cooking time of this additional chicken might differ from that of the other eight chickens.

(ii) Explain how a new line of best fit might differ from that drawn in part (d). *[2 marks]*

3. *[Maximum mark: 18]*

A tent is in the shape of a triangular right prism as shown in the diagram below.

diagram not to scale



The tent has a rectangular base PQRS.
PTS and QVR are isosceles triangles such that PT = TS and QV = VR.
PS is 3.2 m, SR is 4.7 m and the angle TSP is 35°.

(a) Show that the length of side ST is 1.95 m, correct to 3 significant figures. *[3 marks]*

(b) Calculate the area of the triangle PTS. *[3 marks]*

(c) Write down the area of the rectangle STVR. *[1 mark]*

(d) Calculate the **total** surface area of the tent, including the base. *[3 marks]*

(e) Calculate the volume of the tent. *[2 marks]*

A pole is placed from V to M, the midpoint of PS.

(f) Find in metres,

(i) the height of the tent, TM;

(ii) the length of the pole, VM. *[4 marks]*

(g) Calculate the angle between VM and the base of the tent. *[2 marks]*

4. *[Maximum mark: 19]*

On Monday Paco goes to a running track to train. He runs the first lap of the track in 120 seconds. Each lap Paco runs takes him 10 seconds longer than his previous lap.

(a) Find the time, in seconds, Paco takes to run his fifth lap. *[3 marks]*

Paco runs his last lap in 260 seconds.

(b) Find how many laps he has run on Monday. *[3 marks]*

(c) Find the **total** time, in **minutes**, run by Paco on Monday. *[4 marks]*

On Wednesday Paco takes Lola to train. They both run the first lap of the track in 120 seconds. Each lap Lola runs takes 1.06 times as long as her previous lap.

(d) Find the time, in seconds, Lola takes to run her third lap. *[3 marks]*

(e) Find the **total** time, in seconds, Lola takes to run her first four laps. *[3 marks]*

Each lap Paco runs again takes him 10 seconds longer than his previous lap. After a certain number of laps Paco takes less time per lap than Lola.

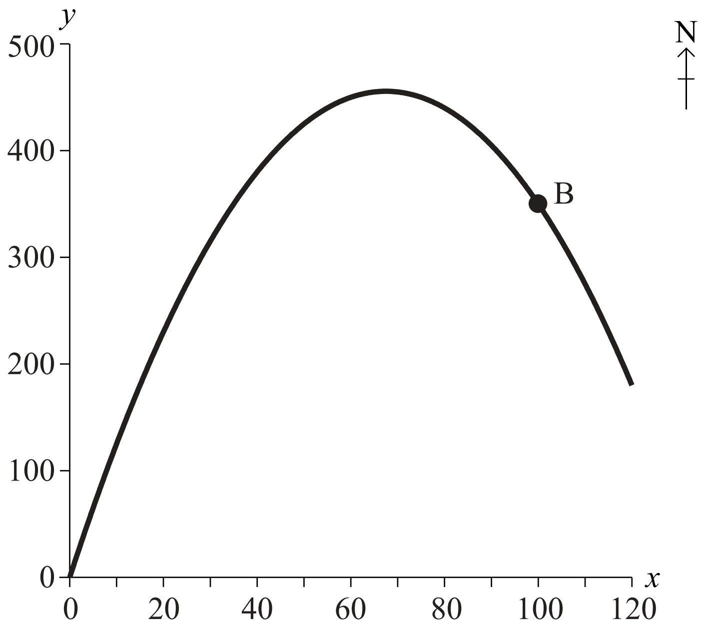
(f) Find the number of the lap when this happens. *[3 marks]*

5. *[Maximum mark: 17]*

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$$y = \frac{-x^2}{10} + \frac{27}{2}x\text{, where } x \geq 0\text{, } y \geq 0$$

(x, y) are the coordinates of a point x metres east and y metres north of O, where O is the origin $(0, 0)$. B is a point on the bicycle track with coordinates $(100, 350)$.



(a) The coordinates of point A are $(75, 450)$. Determine whether point A is on the bicycle track. Give a reason for your answer. *[3 marks]*

(b) Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. *[2 marks]*

(c) Use the answer in part (b) to determine if A $(75, 450)$ is the point furthest north on the track between O and B. Give a reason for your answer. *[4 marks]*

(d) (i) Write down the midpoint of the line segment OB.

(ii) Find the gradient of the line segment OB. *[3 marks]*

(This question continues on the following page)

(Question 5 continued)

Scott starts from a point $\mathrm{C}(0, 150)$. He hikes along a straight road towards the bicycle track, parallel to the line segment OB.

(e) Find the equation of Scott's road. Express your answer in the form $\mathrm{a}x + \mathrm{b}y = \mathrm{c}$, where a, b and $\mathrm{c} \in \mathbb{R}$. *[3 marks]*

(f) Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. *[2 marks]*